4S ACADEMY • IB MATHEMATICS • HL

Paper 3 Strategy: The Ultimate Guide

How to read, structure and score the HL extended-response investigation

Paper 3 is HL only: one hour, two long questions, each building an unfamiliar idea from the ground up. It rewards **mathematical behaviour**, spotting a pattern, forming a conjecture, generalising, and proving, more than recall. This guide shows you how to attack it, and closes with a full worked investigation.

Part 1. How Paper 3 works

what it is really testing

The shape of a Paper 3 question

Key idea. Each question starts easy and gets harder in stages. Early parts are concrete (compute a few cases); middle parts ask you to *conjecture* a general rule; late parts ask you to *prove* or extend it.

Strategy. The early parts are not busywork, they hand you the data you need to see the pattern later. Do them carefully and *write the results in a small table*; the pattern usually jumps out of the table, not the algebra.

Trap. Skipping "trivial" opening parts to save time is the single most costly Paper 3 error, you lose both those marks and the pattern that unlocks everything after.

Reading an unfamiliar context

Strategy. Read the *whole* question before writing anything, so you know where it is heading. New notation is always defined in the question, underline each definition and use it exactly. When a part says "hence", you are meant to use the previous result, not start over.

Use when. "Hence" or "deduce" means reuse the last result. "Otherwise" means a fresh method is allowed. Reading these words correctly saves minutes.

Time and technology

Strategy. Spend about 30 minutes on each question and watch the clock, do not sink 45 minutes into the first. Use your GDC to test conjectures on more cases before you commit to proving them, a conjecture that fails at $n = 5$ is not worth a proof.

Trap. Marks are for *reasoning shown*, not just the final line. Write each step; an unexplained correct answer often scores less than a clearly reasoned one.

Part 2. A worked investigation

pattern, conjecture, proof

The task

Problem. Let $S_n = 1^3 + 2^3 + 3^3 + \cdots + n^3$. (a) Compute S_1, S_2, S_3, S_4. (b) Conjecture a closed formula for S_n. (c) Prove your conjecture.

(a) Gather the data

Solution. $S_1 = 1$, $S_2 = 1 + 8 = 9$, $S_3 = 9 + 27 = 36$, $S_4 = 36 + 64 = 100$. Tabulating: $1, 9, 36, 100$, which are $1^2, 3^2, 6^2, 10^2$, the squares of the triangular numbers $1, 3, 6, 10$.

(b) Conjecture

Solution. The triangular numbers are $T_n = \dfrac{n(n+1)}{2}$, and each S_n is T_n^2. So conjecture $S_n = \left(\dfrac{n(n+1)}{2}\right)^2$.

Check at $n = 4$: $\left(\dfrac{4 \cdot 5}{2}\right)^2 = 10^2 = 100$. ✓

(c) Prove by induction

Solution. Base: $n = 1$: LHS $= 1$, RHS $= \left(\frac{1 \cdot 2}{2}\right)^2 = 1$. ✓

Assume true for $n = k$: $\displaystyle\sum_{i=1}^{k} i^3 = \left(\frac{k(k+1)}{2}\right)^2$.

Step $n = k + 1$: $\displaystyle\sum_{i=1}^{k+1} i^3 = \left(\frac{k(k+1)}{2}\right)^2 + (k+1)^3 = (k+1)^2\left(\frac{k^2}{4} + (k+1)\right) = (k+1)^2 \cdot \frac{k^2 + 4k + 4}{4} = \left(\frac{(k+1)(k+2)}{2}\right)^2.$

This is the formula with $n = k + 1$, so by induction it holds for all $n \in \mathbb{Z}^+$.

Examiner note. A complete induction states the base case, the assumption, the inductive step *using* the assumption, and a closing sentence. The concluding line is a required mark, do not omit it.

4S Academy drills Paper 3 technique on real past investigations, pattern to conjecture to proof, until the structure is automatic. Book a free 15 minute consultation at **4sacademy.com**