4S ACADEMY • IB MATHEMATICS • AA HL

IB Math AA HL: Exam-Style Questions, Fully Worked

Seven original HL questions across the syllabus, with complete method and examiner notes

These are **original 4S Academy practice questions** written in the AA HL style, not a reproduction of any IB paper. Each is worked in full, the way an examiner wants to see it, with a note on where marks are commonly lost. Cover the solution, attempt each first, then check your method.

Part 1. Calculus

differentiation, optimisation, integration

Question 1 • Optimisation

Problem. A closed cylinder has a fixed volume of 1000 cm^3. Find the radius r that minimises its total surface area, and state that minimum area.

Solution. With $V = \pi r^2 h = 1000$, we have $h = \dfrac{1000}{\pi r^2}$. Total surface area $S = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \dfrac{2000}{r}$. Then $\dfrac{dS}{dr} = 4\pi r - \dfrac{2000}{r^2} = 0 \Rightarrow r^3 = \dfrac{500}{\pi} \Rightarrow r = \left(\dfrac{500}{\pi}\right)^{1/3} \approx 5.42$ cm. Since $\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{4000}{r^3} > 0$, this is a minimum. Substituting back, $S \approx 553$ cm^2.

Examiner note. You must justify it is a *minimum* (second derivative > 0, or a sign test), not just solve $S' = 0$. That justification is worth a mark.

Question 2 • Integration

Problem. Find $\displaystyle\int x\, e^{2x}\, dx$.

Solution. Integrate by parts with $u = x$, $dv = e^{2x} dx$, so $du = dx$, $v = \frac{1}{2}e^{2x}$. Then $\displaystyle\int x\, e^{2x}\, dx = \frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\, dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$.

Examiner note. Choose u so that it *simplifies* on differentiating (here $x \to 1$). Getting u and dv the wrong way round leads to a harder integral, not an answer.

Part 2. Algebra and complex numbers

binomial, De Moivre

Question 3 • Binomial theorem

Problem. Find the term independent of x in the expansion of $\left(2x - \dfrac{3}{x^2}\right)^9$.

Solution. The general term is $\binom{9}{k}(2x)^{9-k}\left(-\dfrac{3}{x^2}\right)^k = \binom{9}{k}2^{9-k}(-3)^k\, x^{9-3k}$. Independent of x needs $9 - 3k = 0$, so $k = 3$. The term is $\binom{9}{3}2^6(-3)^3 = 84 \cdot 64 \cdot (-27) = -145152$.

Examiner note. Track the powers of x from *both* brackets: x^{9-k} from the first and x^{-2k} from the second give x^{9-3k}. Forgetting the x^{-2k} is the classic slip.

Question 4 • Complex numbers

Problem. Let $z = 1 + i\sqrt{3}$. Write z in the form $r\,e^{i\theta}$, then find z^6.

Solution. $|z| = \sqrt{1+3} = 2$ and $\arg z = \arctan\dfrac{\sqrt{3}}{1} = \dfrac{\pi}{3}$, so $z = 2e^{i\pi/3}$. By De Moivre, $z^6 = 2^6 e^{i(6\cdot\pi/3)} = 64\,e^{i2\pi} = 64$.

Examiner note. z is in the first quadrant, so $\arg z = \frac{\pi}{3}$ directly. For points in other quadrants, adjust the argument, a blind $\arctan$ gives the wrong angle.

Part 3. Functions, vectors and statistics

the rest of the toolkit

Question 5 • Functions and inverses

Problem. $f(x) = \ln(2x-1)$. State the domain of f, find $f^{-1}(x)$, and state the domain of f^{-1}.

Solution. Domain of f: $2x - 1 > 0 \Rightarrow x > \frac{1}{2}$. To invert, set $y = \ln(2x-1)$, so $e^y = 2x - 1$ and $x = \dfrac{e^y+1}{2}$. Thus $f^{-1}(x) = \dfrac{e^x+1}{2}$. Its domain is the range of f, which is all real numbers, $x \in \mathbb{R}$.

Examiner note. Domain of the inverse $=$ range of the original. State it, do not assume it is all reals without checking.

Question 6 • Vectors

Problem. Line $L_1 : \ \mathbf{r} = (1,2,3) + t(2,-1,2)$ and $L_2 : \ \mathbf{r} = (4,0,1) + s(1,1,-1)$. Find the acute angle between them.

Solution. Use the direction vectors $\mathbf{d_1} = (2,-1,2)$, $\mathbf{d_2} = (1,1,-1)$. Then $\cos\theta = \dfrac{|\mathbf{d_1}\cdot\mathbf{d_2}|}{|\mathbf{d_1}|\,|\mathbf{d_2}|} = \dfrac{|2-1-2|}{3\cdot\sqrt{3}} = \dfrac{1}{3\sqrt{3}} \approx 0.1925$, so $\theta \approx 78.9°$.

Examiner note. Take the *modulus* of the dot product to guarantee the acute angle. Omitting it can give an obtuse angle and lose the final mark.

Question 7 • Normal distribution

Problem. Apple masses X are normally distributed with mean 120 g and standard deviation 15 g. Find $P(X > 140)$, and the mass k below which 90% of apples lie.

Solution. Standardise: $z = \dfrac{140-120}{15} = 1.33$, so $P(X > 140) = P(Z > 1.33) \approx 0.0912$. For the 90% point, $P(Z < z) = 0.90$ gives $z = 1.2816$, so $k = 120 + 1.2816(15) \approx 139$ g.

Examiner note. On the GDC, use the normal cdf for the first part and the inverse normal for the second. Mixing them up is the most common error here.