4S ACADEMY • AP PHYSICS C • ELECTRICITY & MAGNETISM

Gauss's Law: Practice Problem Set

The symmetry method, and four fields derived from scratch, fully worked

Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{A} = \dfrac{Q_{\text{enc}}}{\varepsilon_0}$, turns a hard integral into a one-line calculation *when the charge has symmetry*. The whole skill is choosing a Gaussian surface on which $\mathbf{E}$ is constant in size and either parallel or perpendicular to the surface. These four originals show the method end to end.

The method in three steps

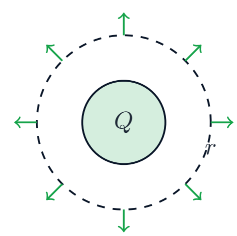
Key idea. (1) Identify the symmetry (spherical, cylindrical, or planar). (2) Draw a Gaussian surface matching it, so $\mathbf{E}$ is constant and normal on the "end" faces and parallel on the rest. (3) Then $E \times (\text{area}) = Q_{\text{enc}}/\varepsilon_0$ and solve for E.

Trap. Gauss's law is always *true*, but only *useful* when symmetry lets you pull E out of the integral. With no symmetry, you must integrate the field directly instead.

1. Field outside a charged sphere

Problem. A sphere carries total charge Q. Find the field at distance r from its centre, for r outside the sphere.

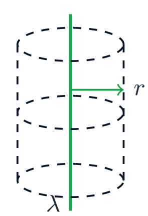
Solution. By spherical symmetry, choose a Gaussian sphere of radius r. $\mathbf{E}$ is radial and constant on it, so $\oint \mathbf{E} \cdot d\mathbf{A} = E(4\pi r^2) = \dfrac{Q}{\varepsilon_0}$, giving $E = \dfrac{Q}{4\pi\varepsilon_0 r^2} = \dfrac{kQ}{r^2}$. The field outside is identical to a point charge Q at the centre.



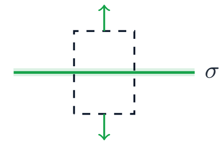
2. Field of an infinite line charge

Problem. A very long line carries charge per unit length λ. Find the field at distance r.

Solution. By cylindrical symmetry, choose a coaxial Gaussian cylinder of radius r and length L. The field is radial, so flux passes only through the curved side: $E(2\pi rL) = \dfrac{\lambda L}{\varepsilon_0}$. Thus $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} = \dfrac{2k\lambda}{r}$. Note $E \propto \frac{1}{r}$, not $\frac{1}{r^2}$.



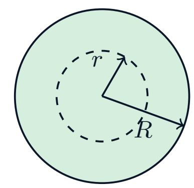
3. Field of an infinite charged plane



Problem. An infinite plane carries surface charge density σ. Find the field near it.

Solution. By planar symmetry, use a "pillbox" straddling the plane with face area A. The field points out of *both* faces, so $E(2A) = \dfrac{\sigma A}{\varepsilon_0}$, giving $E = \dfrac{\sigma}{2\varepsilon_0}$. The field is *uniform*, independent of distance from the plane.

4. Field inside a uniformly charged sphere



Problem. An insulating sphere of radius R has charge Q spread uniformly through its volume. Find the field at $r < R$.

Solution. Only the charge *inside* radius r counts. With uniform density, $Q_{\text{enc}} = Q\dfrac{r^3}{R^3}$. Then $E(4\pi r^2) = \dfrac{Q}{\varepsilon_0}\dfrac{r^3}{R^3}$, so $E = \dfrac{Qr}{4\pi\varepsilon_0 R^3} = \dfrac{kQr}{R^3}$. Inside, E grows *linearly* with r; outside it falls as $1/r^2$.

Remember the shapes. Sphere $\to$ field like a point charge outside ($1/r^2$), linear inside. Line $\to$ $1/r$. Plane $\to$ uniform. Recognising which symmetry you have is 90% of a Gauss's law question.