4S ACADEMY • AP PHYSICS C • MECHANICS

Calculus-Based Mechanics: Final Review

The core equations by topic, each with a worked example and the trap that costs points

Mechanics is a small set of principles applied over and over. This review gives you the key relationships, calculus first, and a worked problem for each, so you revise by *doing*, not just reading.

Part 1. Motion and forces

Kinematics and Newton's second law

Key idea. $v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$, and $\sum \mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$. For constant a: $v = v_0 + at$ and $x = x_0 + v_0 t + \frac{1}{2}at^2$. Friction: $f \le \mu N$.

Problem. A block slides down a $30°$ incline with $\mu = 0.2$. Find its acceleration.

Solution. Along the incline: $ma = mg\sin\theta - \mu mg\cos\theta$, so $a = g(\sin\theta - \mu\cos\theta) = 9.8(0.5 - 0.2 \cdot 0.866) \approx 3.2$ m/s^2.

Trap. Resolve weight into components *along* and *perpendicular* to the incline ($mg\sin\theta$ and $mg\cos\theta$), not into horizontal and vertical. Swapping sine and cosine here is the classic error.

Part 2. Energy and momentum

the conservation laws

Work, energy and power

Key idea. $W = \int \mathbf{F} \cdot d\mathbf{x}$, $K = \frac{1}{2}mv^2$, and $W_{\text{net}} = \Delta K$ (work-energy theorem). Power $P = \frac{dW}{dt} = \mathbf{F} \cdot \mathbf{v}$. Potentials: $U_g = mgh$, $U_{\text{spring}} = \frac{1}{2}kx^2$.

Problem. A 0.5 kg block compresses a spring ($k = 200$ N/m) by 0.1 m and is released on a frictionless surface. Find its launch speed.

Solution. Energy conservation: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = x\sqrt{\frac{k}{m}} = 0.1\sqrt{\frac{200}{0.5}} = 0.1 \cdot 20 = 2$ m/s.

Trap. Use a *consistent* system and account for every energy store. Forgetting friction or a height change is where energy problems go wrong; if a surface is rough, add $-f\,d$.

Momentum and collisions

Key idea. $\mathbf{p} = m\mathbf{v}$, impulse $\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$. Momentum is conserved in every collision; kinetic energy is conserved only in *elastic* ones.

Problem. A 2 kg cart at 3 m/s strikes and sticks to a stationary 1 kg cart. Find their common speed.

Solution. Perfectly inelastic, so $m_1 v_1 = (m_1 + m_2)v'$: $2(3) = (3)v' \Rightarrow v' = 2$ m/s. Note the collision loses kinetic energy (from 9 J to 6 J) even though momentum is conserved.

Trap. "Stick together" means *inelastic*: use momentum only, never energy conservation, to find the final speed.

Part 3. Rotation, oscillations, gravity

Rotational dynamics

Key idea. $\tau = I\alpha$, $L = I\omega$, rotational $K = \frac{1}{2}I\omega^2$. For rolling without slipping, $v = \omega R$. A body rolling down an incline: $a = \dfrac{g\sin\theta}{1 + I/mR^2}$.

Problem. A solid sphere ($I = \frac{2}{5}mR^2$) rolls without slipping down a $30°$ incline. Find its acceleration.

Solution. $a = \dfrac{g\sin\theta}{1 + I/mR^2} = \dfrac{g\sin 30°}{1 + \frac{2}{5}} = \dfrac{g(0.5)}{1.4} = \dfrac{5}{7}g\sin\theta \approx 3.5$ m/s^2.

Trap. A rolling body is slower than a sliding one because energy goes into rotation too. Always include the I/mR^2 term.

Simple harmonic motion

Key idea. $x = A\cos(\omega t + \phi)$. Mass-spring: $\omega = \sqrt{\dfrac{k}{m}}$, $T = 2\pi\sqrt{\dfrac{m}{k}}$. Simple pendulum: $T = 2\pi\sqrt{\dfrac{L}{g}}$. Energy oscillates between $\frac{1}{2}kA^2$ and $\frac{1}{2}mv_{\max}^2$.

Problem. A 0.5 kg mass on a spring ($k = 200$ N/m) oscillates. Find its period.

Solution. $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.5}{200}} = 2\pi(0.05) \approx 0.31$ s. The period is independent of amplitude.

Gravitation

Key idea. $F = \dfrac{Gm_1m_2}{r^2}$, $U = -\dfrac{Gm_1m_2}{r}$ (zero at infinity). Circular orbit: set gravity as the centripetal force, $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r} \Rightarrow v = \sqrt{\dfrac{GM}{r}}$.

Problem. Find the speed of a satellite in a circular orbit of radius r around a planet of mass M.

Solution. From $\dfrac{GMm}{r^2} = \dfrac{mv^2}{r}$, the mass m cancels and $v = \sqrt{\dfrac{GM}{r}}$. Larger orbits are slower.

Trap. Gravitational potential energy is *negative* and uses $1/r$, not mgh. Use mgh only near a surface where g is roughly constant.